

Number Concentration of Cylindrical Particles in a Fluidized Bed*

CAI Jie (蔡杰)^{1,**}, ZHAO Xiaobao (赵孝保)¹, LI Qihe (李奇贺)¹ and YUAN Zhulin (袁竹林)²

¹ School of Energy and Mechanical Engineering, Nanjing Normal University, Nanjing 210042, China

² Thermal-Energy Institute, Southeast University, Nanjing 210096, China

Abstract In this study, a three-dimensional model based on RANS, slender-body theory and Newton-Euler dynamics is established to study the number concentration, one of the most important fluidization characteristics of cylindrical particles. Also, the effects of interaction between cylindrical particles are taken into account by introducing the rigid collision dynamics. To validate the model, the fluidization experiments of cylindrical particles in a cold-state fluidized bed are carried out. The number concentration characteristics of cylindrical particles are obtained from computational fluid dynamics (CFD) simulation. It is found that cylindrical particles arriving at the exit of the riser the earliest come from the near-wall regions, the horizontal transfer of so many cylindrical particles from the radial centre regions to the near-wall regions is evident. Meanwhile, there is no distinct relationship between the number concentration and inlet wind velocity.

Keywords number concentration, cylindrical particles, a three-dimensional model, interaction between cylindrical particles, numerical simulation

1 INTRODUCTION

With the increasing energy demand, more and more attention has been paid to the renewable energy development and utilization. For better use of renewable energy, the fluidized bed combustion technology has been widely applied in energy industry. However, the fluidization of biomass stalks, which decides the combustion efficiency, still remains a challenge. Due to the complex shape features of biomass stalks, the conventional method, such as DEM-CFD (discrete element method-computational fluid dynamics) method, can not describe the fluidization of biomass stalks appropriately, which treats a biomass stalk as a spherical particle and neglect the effects of movement attitude behaviors. Nevertheless, like other fibre suspensions, the attitude behaviors of cylindrical particles affect the fluidization evidently. Therefore, it is necessary to establish a more reasonable model that can describe the rotation behaviors besides translation behaviors in the fluidization accurately for the study of fluidization behaviors of cylindrical particles.

As is known, there are 3 degrees of freedom for the spherical particles while there are 6 degrees of freedom for the cylindrical particles, involving 3 attitude vector parameters besides 3 position vector parameters. Based on the Newton-Euler dynamics [1], the attitude vector of a cylindrical particle can be uniquely positioned by Euler angles, consisting of precession angle (ψ), nutation angle (θ) and spin angle (ϕ). Fig. 1 shows the nutation angle between fixed-body frame and fixed-reference frame.

A few studies on suspension flow of cylindrical particles have been presented. Givler *et al.* [2] proposed

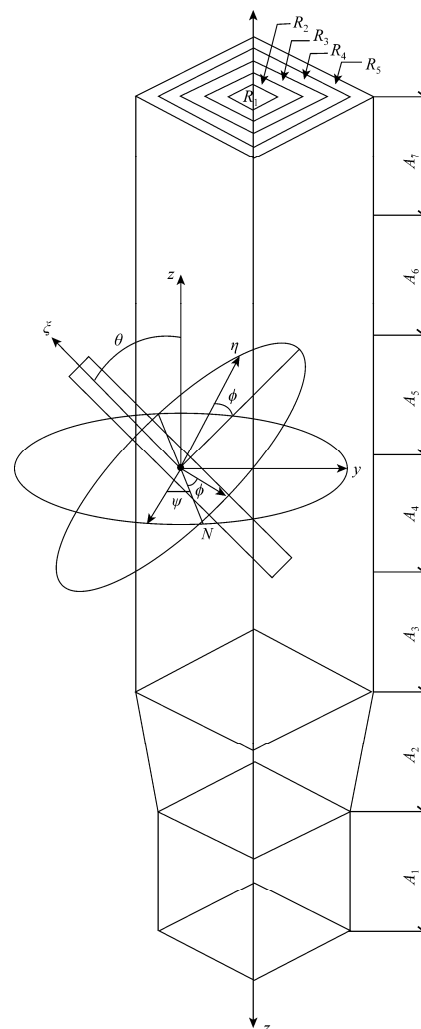


Figure 1 Nutation angle θ between fixed-body frame and the fixed-reference frame

Received 2011-04-06, accepted 2012-09-16.

* Supported by the Natural Science Fund for Colleges and Universities in Jiangsu Province (2011112TSJ0149) and Jiangsu Key Laboratory of Process Enhancement & New Energy Equipment Technology at Nanjing University of Technology, China.

** To whom correspondence should be addressed. E-mail: nnucj@163.com

that fiber orientations can be calculated from the numerical integration of Jeffery's orientation equation along the streamlines obtained on the basis of the finite element method. A mathematical model is proposed by Folgar and Tucker [3] to predict the orientation distribution function of rigid fibers in concentrated suspensions. Butler and Shaqfeh [4] modeled the linear rotational velocities of rigid fibers based on the slender-body theory. Unnikrishnan *et al.* [5–7] did similar experimental investigations of the sedimentation of dilute fiber suspensions. Saintillan *et al.* [8, 9] simulated full-scale suspensions with both periodic and slip boundaries based on a point-particle approximation and on smooth localized force representations. The concentration instability predicted theoretically for suspensions of spheroids had been captured in the simulations. Lin *et al.* [10] pointed out the Stokes number and initial orientation is important parameters which mainly affect the particle motion in a wedge-shaped flow. Zak *et al.* [11] presented a novel two-section based method for statistical characterization of the fiber-orientation distribution within short-fiber composites. Fan *et al.* quantitatively got a correlation of sedimentation of slender particles of large slenderness index through experimental investigations [12]. Fan *et al.* simulated turbulent solid-liquid two-phase flow and orientation of slender particles in a stirred tank with standard Rushton turbines by using the improved inner-outer iterative method and standard k - ε model for the flow turbulence [13]. Cai *et al.* simulated a laminar solid-liquid two-phase flow and orientation of slender particles in a riser [14]. Slender-body theory, Newton-Euler dynamics and rigid collision dynamics is used to describe the translation and rotation of cylindrical particles.

However, as far as we are concerned, the theory research on the fluidization behaviors of cylindrical particles is still at the starting stage. There exist some problems on experimental study of fluidization of cylindrical particles, which may be solved instead by simulation study [15]. In this study, a three-dimensional model based on RANS model, slender-body theory and Newton-Euler dynamics is established to study the number concentration, one of the most important fluidization characteristics of cylindrical particles. Also, the effects of interaction between cylindrical particles are taken into account by introducing the rigid collision dynamics. The present exploration aims for establishing a two-way coupling two-phase flow model of fluidization of cylindrical particles.

2 MATHEMATICAL MODELS

2.1 Gas-phase models

The continuity equation can be written as

$$\frac{\partial(\rho u_j)}{\partial x_j} = 0 \quad (1)$$

Momentum equations are given as

$$\frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x_j} \left[\left(\mu + \mu_t \frac{\partial u_i}{\partial x_j} \right) \right] + \frac{\partial}{\partial x_j} \left[(\mu + \mu_t) \frac{\partial u_j}{\partial x_i} \right] \quad (2)$$

where p is the pressure of flow field, μ is the fluid dynamic viscosity, and μ_t the turbulent viscosity.

The turbulence equations are

$$\begin{aligned} \rho u_j \frac{\partial k}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \mu_t \frac{\partial u_i}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \rho \varepsilon \\ \rho u_j \frac{\partial \varepsilon}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + \mu_t \frac{C_{1\varepsilon} \varepsilon}{k} \frac{\partial u_i}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} \end{aligned} \quad (3)$$

where $\mu_t = \rho C_\mu k^2 / \varepsilon$; C_μ , σ_k , σ_ε , $C_{1\varepsilon}$, $C_{2\varepsilon}$ are turbulent model constants, $C_\mu = 0.09$, $\sigma_k = 1.00$, $\sigma_\varepsilon = 1.30$, $C_{1\varepsilon} = 1.44$, $C_{2\varepsilon} = 1.92$.

Interaction between flow field and cylindrical particles is complicated, which is affected by some factors such as size distribution and concentration of cylindrical particles, characteristics of flow fields, turbulence scales. The force exerted on fluid by solid particles is not considered in this work, namely only one-way coupling between two phases is accounted for. The two-way coupling problem would be targeted in our future study.

2.2 Particle-phase models

2.2.1 Particle-phase models

In order to analyze the drag acting on a cylindrical particle, the cylindrical particle is discretized into 10 parts along its axis based on Gauss quadrature method (Fig. 2). And the drag of 10 discrete parts is analyzed in sequence. Then, the total force acting on the cylindrical particles is obtained by vector summation of all component forces acting on the 10 discrete parts. Meanwhile, the torque acting on the cylindrical particle is calculated according to the rotation around a fixed point theory.

Based on the slender-body theory and the gas-solid two-phase flow theory, the above force analysis algorithm provides a better approach of force and torque along the axis, which results in the translation and rotation of a cylindrical particle in the flow field. The schematic diagram of the force and torque of a discrete part in a flow field is shown in Fig. 2. The space between fluid grid nodes is slightly larger than that between discrete parts. In order to find local flow field

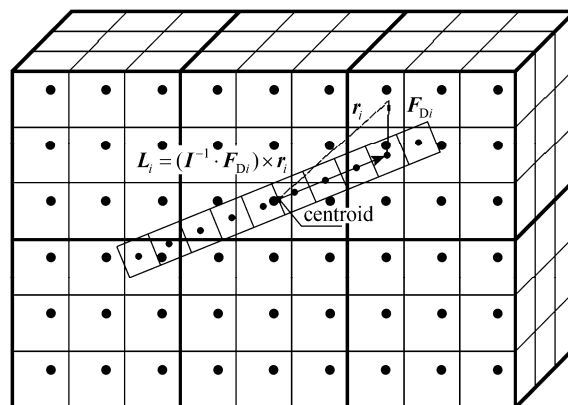


Figure 2 Schematic diagram of force and torque analysis of a discrete part in a flow field

for a discrete part with fewer calculating works, an efficient grid search scheme is designed in this paper [14].

The velocity of discrete part i of a cylindrical particle is

$$\mathbf{u}_i = \mathbf{u} + (\mathbf{I} \cdot \boldsymbol{\omega}) \times \mathbf{r}_i \quad (4)$$

$$\mathbf{I} = \begin{bmatrix} \cos \psi \cos \varphi - \sin \psi \cos \theta \sin \varphi & -\cos \psi \sin \varphi - \sin \psi \cos \theta \cos \varphi & \sin \psi \sin \theta \\ \sin \psi \cos \varphi + \cos \psi \cos \theta \sin \varphi & -\sin \psi \sin \varphi + \cos \psi \cos \theta \cos \varphi & -\cos \psi \sin \theta \\ \sin \theta \sin \varphi & \sin \theta \cos \varphi & \cos \theta \end{bmatrix} \quad (5)$$

The drag acting on discrete part i is given as [16, 17]

$$\mathbf{F}_{Di} = C_D A_p \rho_g |\mathbf{u}_g - \mathbf{u}_i| (\mathbf{u}_g - \mathbf{u}_i) / 2 \quad (6)$$

The drag acting on a cylindrical particle is given as [18, 19]

$$\mathbf{F}_D = \sum_{i=1}^n \lambda_i \mathbf{F}_{Di} \quad (7)$$

The torque acting on a cylindrical particle is given as [1]

$$\mathbf{L} = \sum_{i=1}^n (\mathbf{I}^{-1} \cdot \mathbf{F}_{Di}) \times \mathbf{r}_i \quad (8)$$

where $\mathbf{F}_{Di}$ is the drag acting on discrete part i ; C_D is the fluid drag coefficient; $\mathbf{u}_g(u, v, w)$ is the local fluid velocity; A_p is the sectional area of part i ; ρ_g is the gas density; $\mathbf{u}_i$ is the velocity of discrete part i ; λ_i is the weighted Gauss quadrature coefficient.

Here a model of C_D for cylindrical particles, reported by Fan *et al.* [12], is introduced for error reduction:

$$C_D \cos \phi^* = \frac{24}{Re} (0.006983 + 0.6224 Re^{-1.046}) \times \left(\frac{\rho_p}{\rho_g} \right)^{-1.537} (Ar^*)^{0.8524} \quad (9)$$

where Ar^* is the modified Archimedes number, $Ar^* = d_p^3 (\rho_p - \rho_g)^2 g / \mu^2$; $Re = d_p |\mathbf{u}_g - \mathbf{u}_i| / \nu_g$; $\phi^*(\alpha, \beta, \gamma)$ is the angle between cylindrical particle and the plane

where $\mathbf{u}$ is the centroid velocity of cylindrical particle; $\boldsymbol{\omega}$ is the angular velocity in fixed-body frame; $\mathbf{r}_i$ is the position vector between the discrete part and the centroid of cylindrical particle in fixed-body frame. $\mathbf{I}$ is the cosine matrix of coordinate conversion:

normal to velocity, which can be calculated according to the correlations between the direction cosine and Euler angles,

$$\begin{aligned} \cos\left(\frac{\pi}{2} - \alpha\right) &= \sin \varphi \sin \theta, \\ \cos\left(\frac{\pi}{2} - \beta\right) &= -\cos \psi \sin \theta, \\ \cos\left(\frac{\pi}{2} - \gamma\right) &= \cos \theta \end{aligned} \quad (10)$$

where $(0.5\pi - \alpha)$, $(0.5\pi - \beta)$, $(0.5\pi - \gamma)$ is the angle between $O\xi$ (the axis of cylindrical particle) and Ox , Oy , Oz .

The translation and rotation are tracked according to the Newton-Euler dynamics [1]. Newton-Euler dynamics is the theory base of celestial body dynamics researches and is widely used in aeronautics and astronautics research field. Most of present dynamics models of cylindrical particles are two-dimensional, which may result in some unforeseen loss of effective information of two-phase flow [20]. Some established three-dimensional models lack expansion. Newton-Euler dynamics can analyze the three-dimensional translation and rotation of a celestial body accurately. Moreover, as an independent and complete theoretical system, Newton-Euler dynamics can provide a better platform for further three-dimensional modeling of cylindrical particle suspensions, three-dimensional interaction between cylindrical particles modeling, flexible collision modeling, *etc.*, and for other irregularly shaped particle

suspension modeling. So it is introduced in this work.

Translation equation is given as

$$\frac{d(mu)}{dt} = F_D + g \quad (11)$$

In Eq. (11), g is the gravity acceleration.

The differential equations of the rigid rotation around a fixed point of are expressed as

$$J \cdot \frac{d\omega}{dt} + \omega \times J \cdot \omega = L \quad (12)$$

and its component equations (rotation equations) are given as

$$\begin{cases} J_\xi \dot{\omega}_\xi = L_\xi + (J_\eta - J_\zeta) \omega_\eta \omega_\zeta \\ J_\eta \dot{\omega}_\eta = L_\eta + (J_\zeta - J_\xi) \omega_\zeta \omega_\xi \\ J_\zeta \dot{\omega}_\zeta = L_\zeta + (J_\xi - J_\eta) \omega_\xi \omega_\eta \end{cases} \quad (13)$$

where $\dot{\omega}_\xi$, $\dot{\omega}_\eta$, $\dot{\omega}_\zeta$ is the projection of rotational acceleration $\dot{\omega}$ on the axis of ξ , η , ζ of fixed-body frame; ω_ξ , ω_η , ω_ζ is the projection of ω on the axis of ξ , η , ζ of fixed-body frame; L_ξ , L_η , L_ζ is the projection of L on the axis of ξ , η , ζ of fixed-body frame; J_ξ , J_η , J_ζ is the rotary inertia tensor J of cylindrical particle to the axis of ξ , η , ζ of fixed-body frame.

In order to avoid the singular point in the dynamics calculations, Euler parameters are adopted. The relations between Euler parameters and Euler angles are given as

$$\begin{cases} \lambda_0 = \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{\psi + \phi}{2}\right) \\ \lambda_1 = \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\psi + \phi}{2}\right) \\ \lambda_2 = \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{\psi - \phi}{2}\right) \\ \lambda_3 = \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\psi + \phi}{2}\right) \end{cases} \quad (14)$$

in which ψ is precession angle, θ is nutation angle and ϕ is spin angle. The equations of rotation around a fixed point are expressed as

$$\begin{bmatrix} \dot{\lambda}_0 \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\lambda_1 & -\lambda_2 & -\lambda_3 & \lambda_0 \\ \lambda_0 & -\lambda_3 & \lambda_2 & \lambda_1 \\ \lambda_3 & \lambda_0 & -\lambda_1 & \lambda_2 \\ -\lambda_2 & \lambda_1 & \lambda_0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \omega_\xi \\ \omega_\eta \\ \omega_\zeta \\ 0 \end{bmatrix} \quad (15)$$

$$\begin{cases} \theta' = 2 \times \arccos\left(\sqrt{\lambda_0^2 + \lambda_3^2}\right) \\ \psi' = \arctan\left(\frac{\lambda_3}{\lambda_0}\right) + \arctan\left(\frac{\lambda_2}{\lambda_1}\right) \\ \phi' = \arctan\left(\frac{\lambda_3}{\lambda_0}\right) - \arctan\left(\frac{\lambda_2}{\lambda_1}\right) \end{cases} \quad (16)$$

where ψ' , θ' , ϕ' is the precession angle, nutation angle and spin angle after one time step. The singular point is avoided during coordinate transformation due to the introduction of Euler parameters.

2.2.2 Interaction between cylindrical particles models

For interaction between cylindrical particles analysis, which two cylindrical particles will collide with each other should be determined firstly. In order to reduce the calculation loads, another grid system is established. The unit mesh size of the grid system is 5 times the length of cylindrical particles so as to accommodate enough number of cylindrical particles. In a 3D cell, whether a collision occurs between two cylindrical particles or not is determined according to the position vector between the two cylindrical particles. If the distance between two discrete parts of two cylindrical particles is close enough, they will collide with each other, and the contact point is the two discrete parts. The post-collision velocities and angular velocities of the two colliding cylindrical particles are calculated according to the rigid-body collision dynamics presented by Smith [21].

First, the pre-collision velocities u_i and u_j of collision points of cylindrical particles i and j are given as

$$\begin{cases} u_i = V_{i0} + (I_i \cdot \omega_{i0}) \times r_i = V_{i0} - r_i \times I_i \cdot \omega_{i0} \\ u_j = V_{j0} + (I_j \cdot \omega_{j0}) \times r_j = V_{j0} - r_j \times I_j \cdot \omega_{j0} \end{cases} \quad (17)$$

where V_{i0} , V_{j0} is the pre-collision velocity of the centroid; I_i , I_j is cosine matrix of conversion of coordinate; ω_{i0} , ω_{j0} is pre-collision angular velocity in body axes; r_i , r_j is the relative position vector between collision point and the centroid.

V is the velocity of collision point when it is at the instant of biggest deformation. The component of $u_i - V$ parallel to the line between collision point and the centroid of cylindrical particles i and the component of $u_j - V$ parallel to that between the collision point and the centroid of cylindrical particles j are

$$\begin{cases} (u_i - V)_\parallel = r_{i0} [r_{i0} \cdot (u_i - V)] = P_i \cdot (u_i - V) \\ (u_j - V)_\parallel = r_{j0} [r_{j0} \cdot (u_j - V)] = P_j \cdot (u_j - V) \end{cases} \quad (18)$$

in which r_{i0} , r_{j0} are the unit vector of r_i , r_j ; P_i , P_j are the dyadic tensor of V_{i0} , V_{j0} , respectively. The component of $u_i - V$ perpendicular to the line between the collision point and the centroid of cylindrical particles i and the component of $u_j - V$ perpendicular to line between the collision point and the centroid of cylindrical particles j are expressed as

$$\begin{cases} (u_i - V)_\perp = (u_i - V) - (u_i - V)_\parallel \\ \quad = (\Pi - P_i) \cdot (u_i - V) \\ (u_j - V)_\perp = (u_j - V) - (u_j - V)_\parallel \\ \quad = (\Pi - P_j) \cdot (u_j - V) \end{cases} \quad (19)$$

Then, the following equation was obtained after vector calculation:

$$\begin{cases} (\mathbf{u}_i - \mathbf{V})_{\perp} = \mathbf{V}_{i\perp} + \boldsymbol{\omega}_{i\perp} \times \mathbf{r}_i \\ \quad = \mathbf{V}_{i\perp} - \mathbf{r}_i \times \mathbf{J}_i^{-1} \cdot (\mathbf{r}_i \times m_i \mathbf{V}_{i\perp}) \\ (\mathbf{u}_j - \mathbf{V})_{\perp} = \mathbf{V}_{j\perp} + \boldsymbol{\omega}_{j\perp} \times \mathbf{r}_j \\ \quad = \mathbf{V}_{j\perp} - \mathbf{r}_j \times \mathbf{J}_j^{-1} \cdot (\mathbf{r}_j \times m_j \mathbf{V}_{j\perp}) \end{cases} \quad (20)$$

in which $\mathbf{V}_{i\perp}$, $\mathbf{V}_{j\perp}$ are the velocity variation on the vertical direction of $\mathbf{r}_{i0}$, $\mathbf{r}_{j0}$; $\mathbf{J}_i$, $\mathbf{J}_j$ are the rotary inertia in fixed-reference frame, respectively.

If let

$$\begin{cases} \mathbf{A} = (\mathbf{I} - \mathbf{T}_i)^{-1} \cdot (\mathbf{I} - \mathbf{P}_i) + \mathbf{P}_i \\ \mathbf{B} = (\mathbf{I} - \mathbf{T}_j)^{-1} \cdot (\mathbf{I} - \mathbf{P}_j) + \mathbf{P}_j \\ \mathbf{T}_i = m_i \mathbf{r}_i \times \mathbf{J}_i^{-1} \times \mathbf{r}_i \\ \mathbf{T}_j = m_j \mathbf{r}_j \times \mathbf{J}_j^{-1} \times \mathbf{r}_j \end{cases} \quad (21)$$

where $\mathbf{I}$ is the unit tensor in the fixed-reference frame, the impulses $\mathbf{q}_i$, $\mathbf{q}_j$ are then expressed as

$$\begin{cases} \mathbf{q}_i = m_i \mathbf{A} \cdot [\mathbf{u}_i - (m_i \mathbf{A} + m_j \mathbf{B})^{-1} \cdot (m_i \mathbf{A} \cdot \mathbf{u}_i + m_j \mathbf{B} \cdot \mathbf{u}_j)] \\ \mathbf{q}_j = m_j \mathbf{B} \cdot [\mathbf{u}_j - (m_i \mathbf{A} + m_j \mathbf{B})^{-1} \cdot (m_i \mathbf{A} \cdot \mathbf{u}_i + m_j \mathbf{B} \cdot \mathbf{u}_j)] \\ \mathbf{q}_i + \mathbf{q}_j = 0 \end{cases} \quad (22)$$

Then post-collision velocities $\mathbf{V}_i$, $\mathbf{V}_j$ and the post-collision angular velocity in body axes $\boldsymbol{\omega}_i$, $\boldsymbol{\omega}_j$ are given as

$$\begin{cases} \mathbf{V}_i = \mathbf{V}_{i0} - \frac{1}{m_i} (1 + e) \mathbf{q}_i \\ \boldsymbol{\omega}_i = \boldsymbol{\omega}_{i0} - (1 + 3) \mathbf{J}_i'^{-1} \cdot \mathbf{I}_i^{-1} \cdot (\mathbf{r}_i \times \mathbf{q}_i) \\ \mathbf{V}_j = \mathbf{V}_{j0} - \frac{1}{m_j} (1 + e) \mathbf{q}_j \\ \boldsymbol{\omega}_j = \boldsymbol{\omega}_{j0} - (1 + e) \mathbf{J}_j''^{-1} \cdot \mathbf{I}_j^{-1} \cdot (\mathbf{r}_j \times \mathbf{q}_j) \end{cases} \quad (23)$$

in which the restitution coefficient is $e = 0.5$; $\mathbf{J}_i'$, $\mathbf{J}_i''$ are the rotary inertia in body axes:

$$\begin{cases} \mathbf{I}_i \cdot \mathbf{J}_i'^{-1} \cdot \mathbf{I}_i^{-1} = \mathbf{J}_i^{-1} \\ \mathbf{I}_j \cdot \mathbf{J}_j''^{-1} \cdot \mathbf{I}_j^{-1} = \mathbf{J}_j^{-1} \end{cases} \quad (24)$$

2.2.3 Boundary treatment

Boundary treatment is also based on rigid-body collision dynamics. For wall, m is infinite, $\mathbf{u}_w = 0$, $\boldsymbol{\omega}_w = 0$, $(m_i \mathbf{A} + m \mathbf{B})^{-1} = 0$, so

$$\mathbf{q}_i = m_i \mathbf{A} \cdot \mathbf{u}_i \quad (25)$$

$$\begin{cases} \mathbf{A} = (\mathbf{I} - \mathbf{T}_i)^{-1} \cdot (\mathbf{I} - \mathbf{P}_i) + \mathbf{P}_i \\ \mathbf{B} = (\mathbf{I} - \mathbf{T})^{-1} \cdot (\mathbf{I} - \mathbf{P}) + \mathbf{P} \\ \mathbf{T}_i = m_i \mathbf{r}_i \times \mathbf{J}_i^{-1} \times \mathbf{r}_i \\ \mathbf{T} = m \mathbf{R} \times \mathbf{J}^{-1} \times \mathbf{R} \end{cases} \quad (26)$$

$$\begin{cases} \mathbf{V}_i = \mathbf{V}_{i0} - \frac{1}{m_i} (1 + e) \mathbf{q}_i \\ \boldsymbol{\omega}_i = \boldsymbol{\omega}_{i0} - (1 + e) \mathbf{J}_i'^{-1} \cdot \mathbf{I}_i^{-1} \cdot (\mathbf{r}_i \times \mathbf{q}_i) \end{cases} \quad (27)$$

$$\mathbf{I}_j \cdot \mathbf{J}_i'^{-1} \cdot \mathbf{I}_i^{-1} = \mathbf{J}_i^{-1} \quad (28)$$

2.3 Solution of equations

The fluidization characteristics of cylindrical particles in a gas-solid field, including the effects of interaction between cylindrical particles, are depicted by Eqs. from (1) to (28). Eqs. (13) and (15) are solved with four-order Runge-Kutta algorithm. The discrete phase equations are new models based on Newton-Euler dynamics, rigid-body collision dynamics and slender-body theory, so there are no ready-made source codes, and the source codes are programmed by the authors with C++. The analysis of translation and rotation of cylindrical particles is based on Lagrange method, so there is no need of mesh independence test. FLUENT is a relatively mature flow field simulation software, which is utilized in this work. There are 648000 flow field cells, and the meshes are sparse around the axis of flow field and they are dense in the near-wall region. The gas-solid model in this work is unidirectional coupled, which leads to a separated simulation of gas-phase field and discrete-phase field. The gas-phase field is simulated firstly and the discrete-phase field is simulated based on the gas-phase flow field. The gas-solid two-phase flow is simulated by a double-CPU Dell workstation. CPU is Intel Xeon CPU E5405 2.0 GHz and memory is 3.25 GB. It takes more than 12 h to simulate a gas-phase field. It takes about 3 d to simulate a discrete-phase field.

2.4 Calculation conditions and calculation parameters

The experimental riser consists of three parts: 2 cuboids and a quadrangular frustum between the 2 cuboids. The upper cuboid is 320 mm in side length and 2000 mm in height. The lower cuboid is 260 mm in side length and 500 mm in height. The quadrangular frustum is 320 mm in upper side length, 260 mm in lower side length, and 500 mm in height. The outlet of the experimental riser is open, and the cylindrical particles will be blown out of the riser if they arrive at the outlet. There is a feed inlet on a wall of the lower cuboid, and the sample cylindrical particles will be fed into the riser continually.

The sample cylindrical particles used in the investigations are match sticks of which density are $3.0 \times 10^2 \text{ kg} \cdot \text{m}^{-3}$ and slenderness is between 6 and 12. The initial number of sample cylindrical particles is 500 in the bottom of the riser, and 100 sample cylindrical particles will be uniformly distributed on the bottom of bed every 0.08 s. The three initial Euler

Table 1 Conditions of simulation

$\rho_g/\text{kg}\cdot\text{m}^{-3}$	$v_g/\text{m}^2\cdot\text{s}^{-1}$	$W_{\text{riser}}/\text{m}$	$V_p/\text{m}\cdot\text{s}^{-1}$	$\rho_p/\text{kg}\cdot\text{m}^{-3}$	$\Delta t/\text{s}$	$H_{\text{riser}}/\text{m}$	$\omega_p/\text{rad}\cdot\text{s}^{-1}$
1.205	1.502×10^{-5}	0.32	0	3.0×10^2	1.0×10^{-3}	3	$0-\pi$

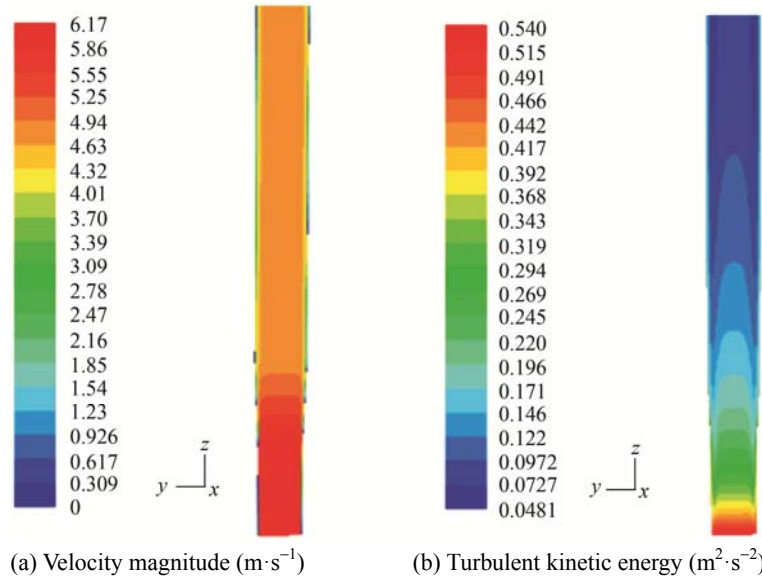


Figure 3 Contours of velocity magnitude ($\text{m}\cdot\text{s}^{-1}$) and turbulent kinetic energy ($\text{m}^2\cdot\text{s}^{-2}$) ($V_{\text{in}} = 6 \text{ m}\cdot\text{s}^{-1}$, $\rho_g = 1.205 \text{ kg}\cdot\text{m}^{-3}$, $v_{\text{air}} = 1.502\times 10^{-5} \text{ m}^2\cdot\text{s}^{-1}$)

angles (θ_p , ψ_p , ϕ_p) are all random number, θ_p is in the range of 0 to π , ψ_p and ϕ_p are in the range of 0 to 2π . The inlet wind velocity V_{in} is between $5.5 \text{ m}\cdot\text{s}^{-1}$ and $7 \text{ m}\cdot\text{s}^{-1}$. Other experiment parameters are given in Tables 1 and 2 respectively.

Table 2 Parameters of simulation

	$\theta/(\circ)$	A_i/m	R_i/m	κ	$V_{\text{in}}/\text{m}\cdot\text{s}^{-1}$
1	0–15	0–0.5	0–0.064	6	5.5
2	15–30	0.5–1.0	0.064–0.128	8	6
3	30–45	1.0–1.4	0.128–0.192	10	6.5
4	45–60	1.4–1.8	0.192–0.256	12	7
5	60–75	1.8–2.2	0.256–0.32		
6	75–90	2.2–2.6			
7		2.6–3.0			

3 RESULTS AND DISCUSSION

3.1 Simulation photos of the fluidization of cylindrical particles

Figure 3 shows the contours of velocity magnitude and turbulent kinetic energy. It can be seen that there is an evident velocity gradient along the axial direction and radial direction in the riser and the velocity grads in near-wall region is more evident, and there is an evident region distribution difference of

turbulent kinetic energy. The local fluid velocity in the upper cuboid is lower than that in the lower cuboid because the sectional area of the lower cuboid is smaller than that of the upper cuboid.

Figure 4 shows the fluidization of cylindrical particles of 4 different slendernesses with the number ratio of 1 : 1 : 1 : 1. 4 colors represent 4 slendernesses of cylindrical particles respectively. The slenderness of blue cylindrical particles is 6 and that of red cylindrical particles is 12. Fig. 5 shows the experimental photos of the fluidization of cylindrical particles in A_2 and A_3 . The axial number concentration of cylindrical particles in the riser during the fluidization is shown in Fig. 6. It can be seen that number concentration of cylindrical particles increases with the height in the riser and it approaches the peak value in A_6 and then decreases in the exit area.

In the fluidized state, there are about 6000–8000 cylindrical particles in the riser. Some cylindrical particles with bigger slendernesses can not be blown out of the riser by air flow because of bigger mass, so the number of cylindrical particles in the fluidized bed is increasing with the growing simulation time steps.

Figure 7 shows the simulated results of time consumption of cylindrical particles arriving at the outlet the earliest in different radial zones. We can see that the cylindrical particles arriving at the exit of the riser the earliest come from the near-wall regions.

Number concentration is the number percentage of cylindrical particle number in a zone of a riser to the sum of cylindrical particles in the riser, and the

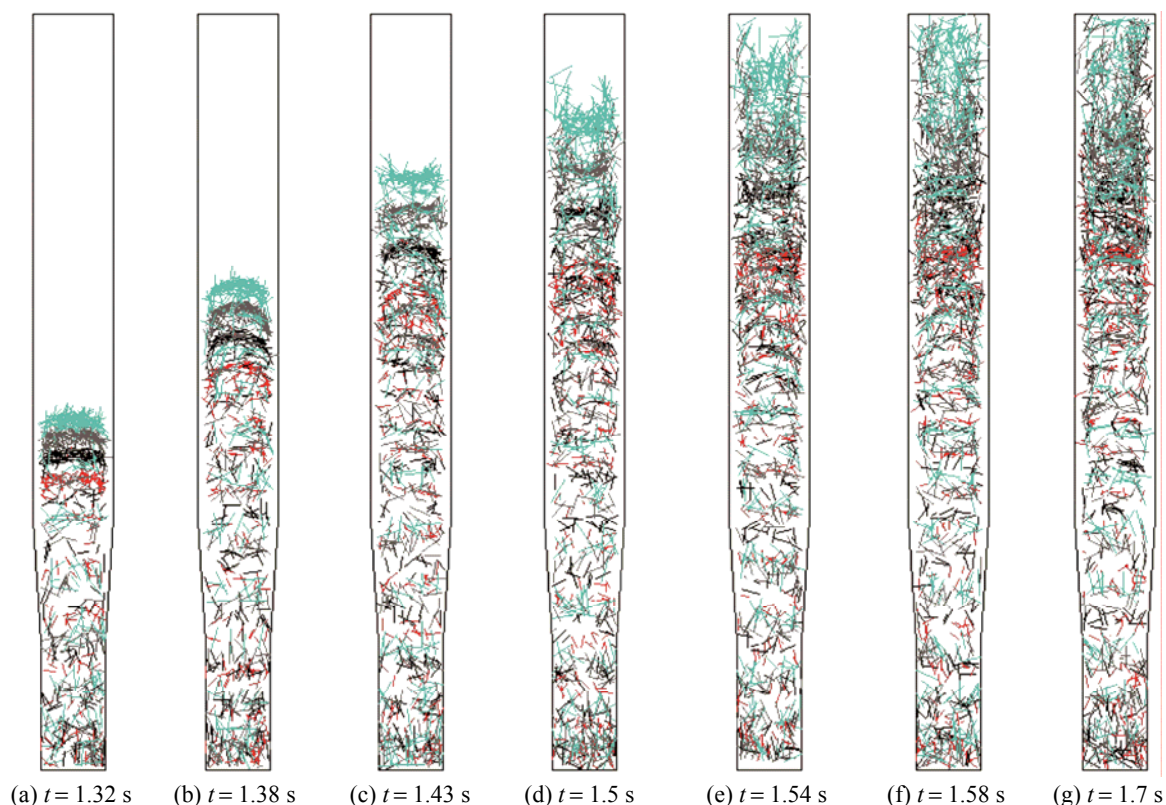


Figure 4 Fluidization simulation of cylindrical particles in the riser ($V_{in} = 6 \text{ m} \cdot \text{s}^{-1}$, $V_p = 0 \text{ m} \cdot \text{s}^{-1}$, $\rho_p = 3.0 \times 10^2 \text{ kg} \cdot \text{m}^{-3}$, $\theta_{in} = 0^\circ - 180^\circ$, $A_1 - A_7$, $R_1 - R_5$, $\kappa = 6 - 12$)

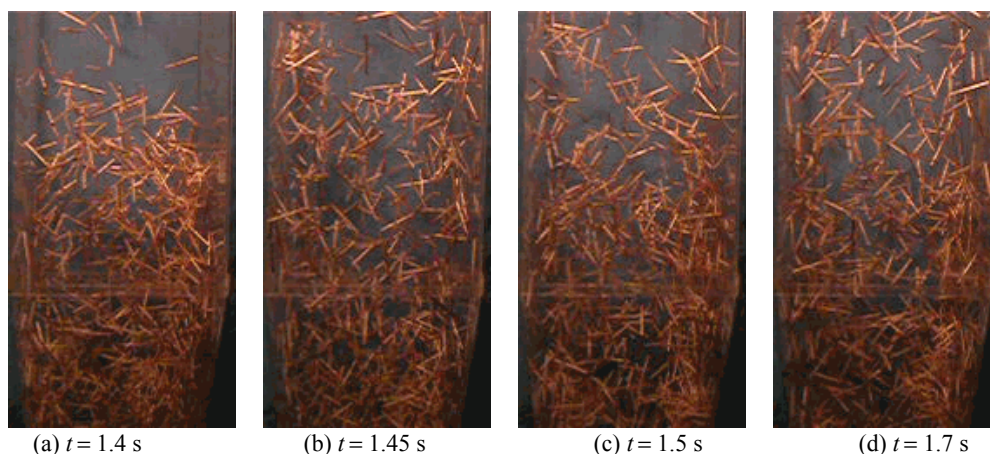


Figure 5 Experimental photos of the fluidization of cylindrical particles in A_2 and A_3 ($V_{in} = 6 \text{ m} \cdot \text{s}^{-1}$, $V_p = 0 \text{ m} \cdot \text{s}^{-1}$, $\rho_p = 3.0 \times 10^2 \text{ kg} \cdot \text{m}^{-3}$, $\theta_{in} = 0^\circ - 180^\circ$, $A_1 - A_7$, $R_1 - R_5$, $\kappa = 6 - 12$)

norm is the sum of cylindrical particles in the riser. Number concentration of cylindrical particles in the riser at the moment of 2 s is shown in Fig. 8. It is found that number concentration is uneven in different areas in the riser and perhaps, there are some particle clusters. The occurrence of particle clusters during the fluidization is due to interaction between cylindrical particles, and it is necessary to establish a model of inter-cylindrical particles collision. So the establishment of a 3D model of inter-cylindrical particles colli-

sion based on rigid-body collision dynamics is beneficial for further studying the true nature of clustering of cylindrical particles.

3.2 Axial profile of number concentrations

Figure 9 shows the simulated results of number concentration of cylindrical particles in different axial regions in the riser. They are the average values of 5 time

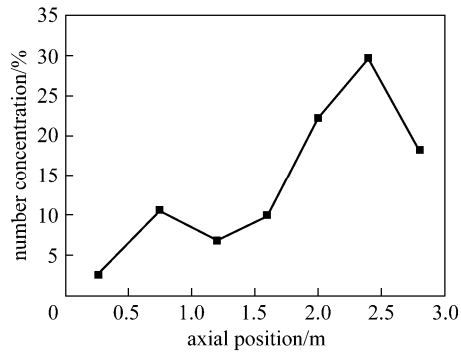


Figure 6 Axial concentration of cylindrical particles in the riser ($V_{in} = 6 \text{ m}\cdot\text{s}^{-1}$, $V_p = 0 \text{ m}\cdot\text{s}^{-1}$, $\rho_p = 3.0 \times 10^2 \text{ kg}\cdot\text{m}^{-3}$, $n_{\kappa_1} : n_{\kappa_2} : n_{\kappa_3} : n_{\kappa_4} = 1:1:1:1$, $\theta_{in} = 0^\circ-180^\circ$, A_1-A_7 , R_1-R_5 , $\kappa = 6-12$)

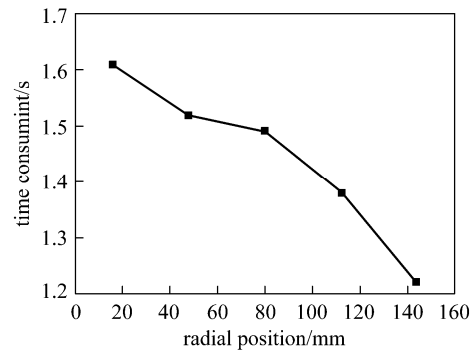


Figure 7 Time consumption for the earliest of cylindrical particles to arriving at the outlet in different radial zones ($V_{in} = 6 \text{ m}\cdot\text{s}^{-1}$, $V_p = 0 \text{ m}\cdot\text{s}^{-1}$, $\rho_p = 3.0 \times 10^2 \text{ kg}\cdot\text{m}^{-3}$, $\theta_{in} = 0^\circ-180^\circ$, A_1-A_7 , R_1-R_5 , $\kappa = 6-12$, $n_{\kappa_1} : n_{\kappa_2} : n_{\kappa_3} : n_{\kappa_4} = 1:1:1:1$)

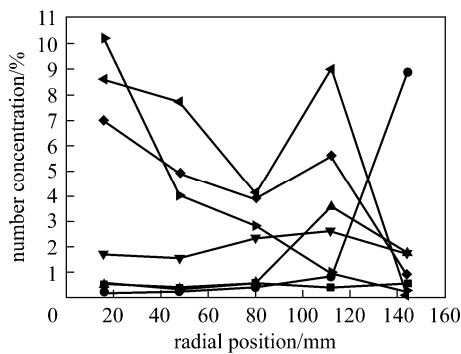


Figure 8 Number concentration of cylindrical particles in the riser at time of 2 s ($V_{in} = 6 \text{ m}\cdot\text{s}^{-1}$, $V_p = 0 \text{ m}\cdot\text{s}^{-1}$, $n_{\kappa_1} : n_{\kappa_2} : n_{\kappa_3} : n_{\kappa_4} = 1:1:1:1$, $\rho_p = 3.0 \times 10^2 \text{ kg}\cdot\text{m}^{-3}$, $\theta_{in} = 0^\circ-180^\circ$, A_1-A_7 , R_1-R_5 , $\kappa = 6-12$)
 ■ A_1 ; ● A_2 ; ▲ A_3 ; ▼ A_4 ; ◆ A_5 ; ◀ A_6 ; ▶ A_7

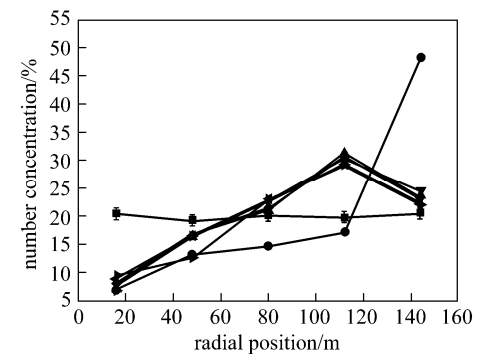


Figure 9 Radial number concentrations of cylindrical particles in different axial regions ($V_{in} = 6 \text{ m}\cdot\text{s}^{-1}$, $n_{\kappa_1} : n_{\kappa_2} : n_{\kappa_3} : n_{\kappa_4} = 1:1:1:1$, $V_p = 0 \text{ m}\cdot\text{s}^{-1}$, $\rho_p = 3.0 \times 10^2 \text{ kg}\cdot\text{m}^{-3}$, $\theta_{in} = 0^\circ-180^\circ$, A_1-A_7 , R_1-R_5 , $\kappa = 6-12$)
 ■ A_1 ; ● A_2 ; ▲ A_3 ; ▼ A_4 ; ◆ A_5 ; ◀ A_6 ; ▶ A_7

steps, so having significance of statistics. These regions have been identified in Fig. 1 and Table 2. As shown in Fig. 9, the disparity of number concentration in A_1 is not distinct owing to the fixed original number ratios of cylindrical particles of 4 slendernesses. In A_2 , many cylindrical particles gather in the near-wall region as a result of the evident local flow radial velocity gradient. Above A_2 , number concentration of cylindrical particles in the near-wall regions is always much bigger than that in the radial centre. As is known, spherical particles will move from the radial centre to the near-wall regions during the fluidization because the wind velocity in the radial centre is bigger than that in near-wall regions. Similarly, many cylindrical particles will move from the radial centre to the near-wall regions during the fluidization of slender particles. This result that the number concentration of cylindrical particles in radial centre zones is much bigger than that in near-wall zones is consistent with the conclusion presented by Olson *et al* [22].

3.3 Orientation of cylindrical particles

Orientation is an important attitude behavior of

fluidized cylindrical particles. Fig. 10 (a) shows the mutation angles of cylindrical particles in different axial zones in the riser. In region A_1 , there is no evident orientation, and the number proportions of cylindrical particles moving in different nutation angle ranges are nearly the same, which is owing to the bed bottom effect. Fig. 10 (b) shows the simulated and experimental results of orientation in the axial direction and the simulated results of orientation in the x-axis and y-axis. It is found that the orientations in the x- and y-axes are not evident. The phenomenon can be explained by the principle of minimum potential energy: if a system has the lowest potential energy, the system will be in a steady state. In order to get over the effects of velocity gradient in the flow field and keep stable movement attitude, the cylindrical particles need to move in small nutation angles, *i.e.*, particles positioned almost horizontally.

3.4 The effects of inlet wind velocity on radial number concentrations

Figure 11 shows the radial profile of number concentration of cylindrical particles from A_3 to A_7 at

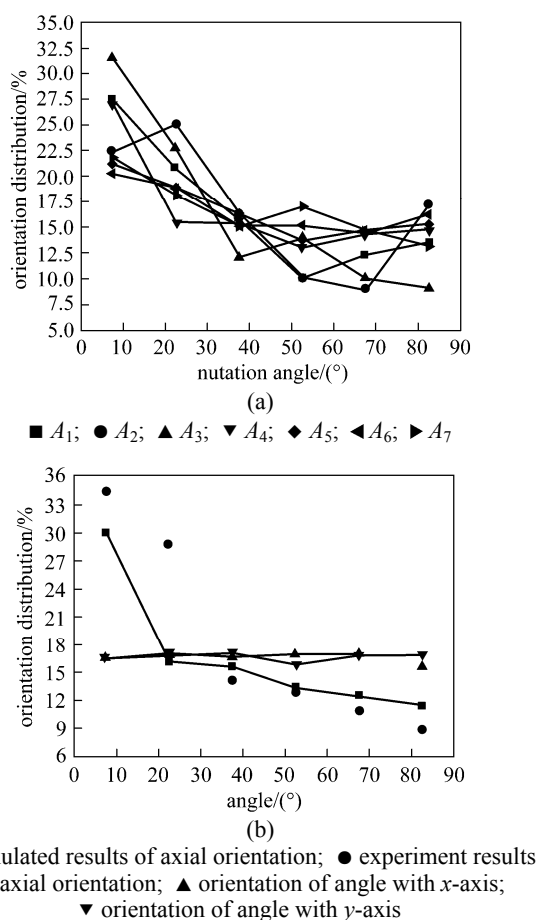


Figure 10 Orientation of cylindrical particles in different axial zones in the riser ($V_{in} = 6 \text{ m}\cdot\text{s}^{-1}$, $V_p = 0 \text{ m}\cdot\text{s}^{-1}$, $\rho_p = 3.0 \times 10^2 \text{ kg}\cdot\text{m}^{-3}$, $\theta_{in} = 0^\circ\text{--}180^\circ$, $A_1\text{--}A_7$, $R_1\text{--}R_5$, $\kappa = 6\text{--}12$, $n_{k_1} : n_{k_2} : n_{k_3} : n_{k_4} = 1:1:1:1$)

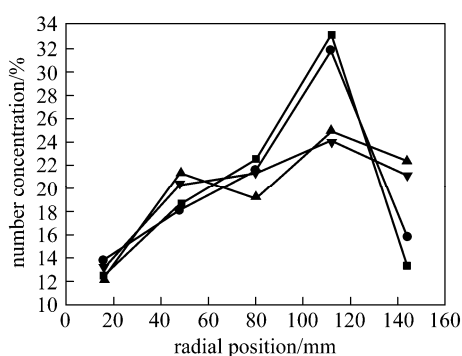


Figure 11 Radial number concentrations of cylindrical particles at different inlet wind velocities ($V_{in} = 6 \text{ m}\cdot\text{s}^{-1}$, $V_p = 0 \text{ m}\cdot\text{s}^{-1}$, $\rho_p = 3.0 \times 10^2 \text{ kg}\cdot\text{m}^{-3}$, $\theta_{in} = 0^\circ\text{--}180^\circ$, $A_1\text{--}A_7$, $R_1\text{--}R_5$, $\kappa = 6\text{--}12$, $n_{k_1} : n_{k_2} : n_{k_3} : n_{k_4} = 1:1:1:1$) $V/\text{m}\cdot\text{s}^{-1}$: ■ 5.5; ● 6; ▲ 6.5; ▼ 7

different inlet wind velocities. Cylindrical particles are in the fluidized state at the inlet wind velocity of 5 and $6 \text{ m}\cdot\text{s}^{-1}$, while they are in the pneumatic conveying state when the inlet wind velocity is over $6 \text{ m}\cdot\text{s}^{-1}$. The number concentrations of cylindrical particles in the

pneumatic conveying state are more even than those in the fluidized state. In both the fluidized state and pneumatic conveying state, the number concentrations of cylindrical particles don't change evidently with different inlet wind velocities. Maybe there is little influence of the lateral velocity gradient on the motion of cylindrical particle cluster from radial centre zone to the near-wall region. The number concentration of cylindrical particles in the near-wall region is the biggest, while it is the lowest in the center region. In the fluidized state, there is no evident relationship between radial number concentration and inlet wind velocity.

3.5 Effect of slenderness on axial number concentration

As is known, for the fluidization of spherical particles, wide particle size distribution is the reason of the formation of evident dense phase zone and suspension section in the riser. Here the effects of slenderness distribution on the formation of dense phase zone and suspension section in the riser is studied as shown in Fig. 12. The number concentration of cylindrical particles of smaller slenderness decreases firstly with the axial position until up to the minimum value, and then increases quickly, on the contrary the number concentration of cylindrical particles of bigger slenderness increases with the height and it arrives at the maximum value in certain axial height and then decreases quickly. We also can see from Fig. 5 that so many red cylindrical particles stay in A_4 and A_5 . The reason is that cylindrical particles of smaller slenderness can pass through the riser but cylindrical particles of bigger slenderness can not fly out the riser and most of them will stay in some axial zone due to their larger mass. The height of dense phase zone is too high when the inlet velocity is $6 \text{ m}\cdot\text{s}^{-1}$ and slenderness distribution is $1:1:1:1$. It is found that there are evident effects of slenderness distribution on the formation of dense phase zone and suspension section in a CFD, bigger slenderness distribution of cylindrical particles is necessary and the ratio between the maximal

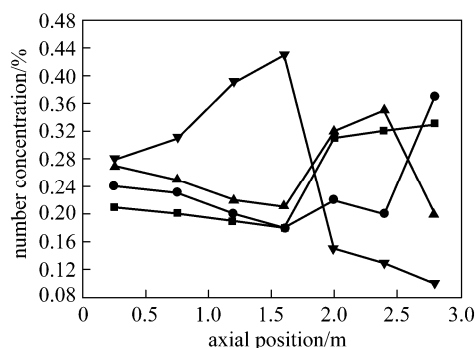


Figure 12 Number concentration of different slenderness in different axial zones ($V_{in} = 6 \text{ m}\cdot\text{s}^{-1}$, $V_p = 0 \text{ m}\cdot\text{s}^{-1}$, $n_{k_1} : n_{k_2} : n_{k_3} : n_{k_4} = 1:1:1:1$, $\rho_p = 3.0 \times 10^2 \text{ kg}\cdot\text{m}^{-3}$, $\theta_{in} = 0^\circ\text{--}180^\circ$, $A_1\text{--}A_7$, $R_1\text{--}R_5$, $\kappa = 6\text{--}12$)

■ $k_1 = 6$; ● $k_2 = 8$; ▲ $k_3 = 10$; ▼ $k_4 = 12$

slenderness and the minimal slenderness must be much more than 2.

4 CONCLUSIONS

In this paper a 3D gas-solid two-phase flow model of the fluidization of cylindrical particles is established to study the number concentration characteristics of cylindrical particles. The main simulation results are shown as follows:

(1) The cylindrical particles arriving at the exit of the riser the earliest come from the near-wall regions.

(2) There is an evident horizontal transfer of some cylindrical particles from the radial centre regions to the near-wall regions during the fluidization, which is similar to the fluidization of spherical particles.

(3) In the fluidized state, there is no evident relationship between radial number concentration and inlet wind velocity.

(4) Most cylindrical particles move with attitude of being approximately parallel to the axis of the riser during the fluidization.

(5) There are evident effects of slenderness distribution on the formation of dense phase zone and suspension section in a CFD, bigger slenderness distribution of cylindrical particles is necessary and the ratio between the maximal slenderness and the minimal slenderness must be much more than 2.

NOMENCLATURE

A_i	axial region distribution in the riser, m
H_{riser}	height of riser, m
R_i	radial region distribution in the riser, m
Δt	time step, s
V_{in}	wind velocity of entrance, $\text{m}\cdot\text{s}^{-1}$
V_p	initial velocity of cylindrical particle, $\text{m}\cdot\text{s}^{-1}$
θ_p	nutation angle, ($^\circ$)
κ	slenderness of cylindrical particle
ρ_g	air density, $\text{kg}\cdot\text{m}^{-3}$
ρ_p	stalk density, $\text{kg}\cdot\text{m}^{-3}$
ν_g	kinematic viscosity of gas, $\text{m}^2\cdot\text{s}^{-1}$
ω_p	initial angular velocity of cylindrical particle, $\text{rad}\cdot\text{s}^{-1}$

REFERENCES

- Ardema, M.D., *Newton-Euler Dynamics*, Springer, California, USA (2005).
- Givler, R.C., Crochet, M.J., Pipes, R.B., "Numerical prediction of fiber orientation in dilute suspensions", *J. Compos. Mater.*, **17** (4), 330–343 (1983).
- Folgar, F.P., Tucker, C.L., "Orientation behavior of fibers in concentrated suspensions", *J. Reinforc. Plast. Compos.*, **3** (2), 98–119 (1984).
- Butler, J.E., Shaqfeh, E.S.G., "Dynamic simulation of the inhomogeneous sedimentation of rigid fibers", *J. Fluid Mech.*, **468**, 205–237 (2002).
- Unnikrishnan, A., Chhabra, R.P., "An experimental study of motion of cylinders in Newtonian fluids—Wall effects and drag coefficient", *Can. J. Chem. Eng.*, **69**, 729–734 (1991).
- Herzhaft, B., "Experimental investigation of the sedimentation of a dilute fiber suspension", *Phys. Rev. Lett.*, **77** (2), 290–301 (1996).
- Metzger, B., Butler, J.E., Guazzelli, E., "Experimental investigation of the instability of a sedimenting suspension of fibres", *J. Fluid Mech.*, **575**, 307–332 (2007).
- Saintillan, D., Shaqfeh, E.S.G., Darve, E., "The growth of concentration fluctuations in dilute dispersions of orientable and deformable particles under sedimentation", *J. Fluid Mech.*, **53**, 347–388 (2006).
- Saintillan, D., Darve, E., Shaqfeh, E.S.G., "A smooth particle-mesh Ewald algorithm for Stokes suspension simulations: The sedimentation of fibres", *Phys. Rev. Lett.*, **99** (5), 58–72 (2005).
- Lin, J.Z., Liu, Z.Q., Wang, Y.L., "Research on the motion characteristics of fiber suspensions in a wedge-shaped flow", *J. Hydrodyn.*, **12** (2), 92–100 (2000).
- Zak, G., Park, C.B., Benhabib, B., "Estimation of three-dimensional fiber-orientation distribution in short-fiber composites by a two-section method", *J. Compos. Materials*, **35** (4), 316–339 (2001).
- Fan, L., Mao, Z.S., Yang, C., "Experiment on settling of slender particles with large aspect ratio and correlation of drag coefficient", *Ind. Eng. Chem. Res.*, **43** (23), 7664–7670 (2004).
- Fan, L., Mao, Z.S., Wang, Y.D., "Numerical simulation of turbulent solid-liquid two-phase flow and orientation of slender particles in a stirred tank", *Chem. Eng. Sci.*, **60**, 7045–7056 (2005).
- Cai, J., Li, Q.H., Yuan, Z.L., "Orientation of cylindrical particles in gas-solid circulating fluidized bed", *Particuology*, **6** (1), 89–96 (2012).
- Alexander, K., Efstathios, E.M., "An analytical approach for the closure equations of gas-solid flows with inter-particle collisions", *Int. J. Multiphase Flow*, **30** (2), 159–180 (2004).
- Yuan, Z.L., "Study on fluidized region of particles using direct simulation method", *J. Combust. Sci. Technol.*, **7** (2), 120–122 (2001).
- Tsuji, Y., Tanaka, T., Yonemura, S., "Cluster patterns in circulating fluidized beds predicted by numerical simulation (discrete particle model versus two-fluid model)", *Powder Technol.*, **95**, 254–264 (1998).
- Mackaplow, M.B., Shaqfeh, E.S.G., "A numerical study of the rheological properties of suspensions of rigid, non-Brownian fibres", *J. Fluid. Mech.*, **329**, 155–176 (1996).
- Paschkewitz, J.S., Dubief, Y., Dimitropoulos, C.D., Shaqfeh, E.S.G., Parviz, M., "Numerical simulation of turbulent drag reduction using rigid fibres", *J. Fluid Mech.*, **518** (10), 281–317 (2004).
- Shikha, T.S., "Vertical axis wind rotor with concentration by convergent nozzles", *Wind. Eng.*, **27** (6), 555–559 (2003).
- Smith, C.E., "Predicting rebounds using rigid-body dynamics", *Trans. ASME J. Appl. Mech.*, **58**, 754–758 (1991).
- Olson, J.A., Frigaard, I., Chan, C., "Modeling a turbulent fibre suspension flowing in a planar contraction: The one-dimensional head-box", *Int. J. Multiphase Flow*, **30**, 51–66 (2004).